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be useful to others to know beforehand what to look for if there is so conspicuous an effect as to suggest mechanical correction.

2. The remark of the Astronomer-Royal is briefly to the effect that an error of *centring* of one lens, *i.e.* a lateral displacement of one lens relatively to the other, is not so likely to produce serious effects as a relative *tilt*. This he found by personal experience when adjusting the 28-inch objective at the Royal Observatory. There was originally a defect which gave unsymmetrical out-of-focus images, and he in the first instance attributed it to defective centring as above, but found on trial that one lens could be moved laterally with reference to the other by quantities of the order of a quarter of an inch without serious change of the images. He then tried relative tilt, and found that a very small correction of that kind gave the required results. Hence it seems more likely that we should look for defects in this direction, especially if the lenses of an objective are sensibly separated.

3. The defect in the Algiers objective, which gave rise to the well-marked magnitude-equation, was apparently of this kind, though arising in rather a different way. I gather from M. Trépied that a representative from M. Gautier found the lenses of the objective pressed too tightly together. Such pressure would be very unlikely to be quite symmetrical, and the effect might be regarded as a form of tilt.

4. With heavy objectives it seems desirable to be on the watch for an error of this kind depending on the pressure produced by the weight of the objective itself. Such errors would, it is to be hoped, be very small; but they might come within the scope of very refined investigations; and they might unfortunately become serious from the enormous labour required to investigate them. They would, in each individual plate, be associated with "driving error," or correction-of-refraction error which might be of the same kind; and could only be determined from the mean of a large number of plates.

On the Possible Effects of Radiation on the Motion of Comets, with special reference to Encke's Comet. By H. C. Plummer, M.A.

1. It may be said with confidence that the theory of radiation, which occupies a prominent place in modern physics, possesses an importance in astronomy which is only beginning to be appreciated. It has been suggested that the repulsive force manifested in the tails of comets finds its simplest explanation in this theory, and quite recently Arrhenius has appealed to the properties of radiation in a discussion of the physical nature of the solar corona. To Professor Poynting we are indebted for a memoir on

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"Radiation in the Solar System" (published in the *Philosophical Transactions* and reprinted in the *Memoirs* of this Society), which supplies a lucid exposition of those details of the subject which mainly concern the astronomer. The discussion of the effects of radiation pressure, which is contained in the second part of his paper, suggests many points of great astronomical interest, and may well lead us to reconsider some of the conclusions hitherto held. Naturally it is in the sphere of meteoric astronomy that the influence is mainly to be expected. To take an example, two spherical meteors of density 6·2 and radius 1 cm., possessing the temperature which they acquire at the same distance from the Sun as the Earth, neither attract nor repel one another, the gravitational stress being balanced by the radiation pressure. Facts such as this may well lead to a revision of the views at present held as to the stability of meteor swarms, the subject of a series of well-known researches by Schiaparelli, Charlier, L. Picart and Callandreau. As Poynting suggests, such facts may have a bearing on the theory of Saturn's rings provided the temperature of the planet is sufficiently high. Most significant of all, from the theoretical point of view, is the modification, though not of course the destruction in the physical sense, of the law of action and reaction. It seems just conceivable that the transformation of material momentum into ether-momentum may have important influences on a cosmic scale.

2. If the meteoric hypothesis as to the constitution of comets be accepted, the results which apply to meteors can be extended to comets. The degree of concentration of the meteors must be sufficient to account for the luminosity of the comet and yet not so great but that each meteor may have a practically unobstructed view of the Sun. For the sake of simplicity it is assumed that the meteors are spherical in shape, and that they are perfect absorbers of radiation, though this cannot be the case. If r is the distance from the Sun, the gravitation pull may be denoted by μ/r^2 and the radiation pressure by μ'/r^2. Using the formula and constants given by Poynting (p. 540*), we have

$$\frac{\mu'}{\mu} = \frac{7{\cdot}4 \times 10^{-5}}{a\rho}$$

where a is the radius of the meteoric particle and ρ its density, the C.G.S. system of units being adopted. Now if a is the mean distance and n the mean motion of the particle, the expression of Kepler's third law is

$$n^2a^3 = \mu$$

where μ is a constant depending only on the mass of the Sun and on the system of fundamental units employed. But when allowance is made for the effect of radiation pressure the law is modified so that its expression becomes

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* The reference here and in § 5 is to *Phil. Trans.* vol. 202 A.

$$n^2a^3 = \mu - \mu'$$

The change implies a virtual reduction in the Sun's mass. If μ'/μ is one-millionth and ρ is taken to be about 5·5, the corresponding value of a is about 13 cm. If the size of the constituent meteors reaches or exceeds this limit it is scarcely likely, with the present degree of accuracy in comet observations, that the departure from Kepler's law can be detected. But the meteors may very easily be considerably less than 10 inches in diameter, and in that case it should be possible to find evidence of the action of radiation pressure.

3. There is no difficulty in finding discrepancies between theory and observation in the orbital motions of comets, and the important question which confronts us is this : How far can such discrepancies be explained by the fact that an incorrect value may have been assumed of the effective mass of the Sun ? The intention here is simply to suggest this question, not to give any definite answer to it. The case which has received the greatest amount of study is that of Encke's comet. The elements calculated for one apparition and corrected for the perturbations caused by known sources of disturbance during the interval in which the comet is beyond the range of observation are not satisfied by the observations made at the following apparition. To meet the difficulty it has been usual, from the time of Encke, to assign an empirical variation to the mean motion. This is explained by the hypothesis of a resisting medium for which there is no confirmatory evidence, and which seems to have been introduced simply *ad hoc.* Moreover, since the variation of the mean motion has not remained constant in all revolutions of the comet, it has been necessary to attribute properties to the medium which seem rather artificial. However this may be, the relation connecting the mean distance and the mean motion (thus varied) with what may be called the planetary mass of the Sun has presumably been preserved. If this assumption has in fact been made it is an important question how far the discrepancies between theory and observation can be explained by making allowance for the pressure due to solar radiation. Possibly a negative answer is implicitly contained in those works to which Dr. Backlund has devoted so much skill and labour. But if not, an interesting though laborious research is suggested, which has not yet been undertaken. The point here insisted on is that in the case of all comets it is unwarrantable to *assume* that the mean motion and the mean distance are related and not independent elements.

4. It has been supposed in what precedes that the size of the particles of which a comet is composed is uniform and permanent. This may not be the case. Collisions may cause them to coalesce,

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and so to increase progressively in size. The radiation pressure will thus be diminished and an acceleration of the mean motion may result from this cause. At the same time there will be a corresponding diminution in the brightness of the comet. It is easily seen, in fact, that if m spherical particles coalesce into n meteors both the radiation pressure, expressed in terms of the corresponding gravitative pull, and the brightness of the comet are changed in the same ratio—namely, the cube root of $m : n$. Other causes may be imagined for an alteration, either continuous or discontinuous, in the size of the meteors. Hence, even if their size is supposed originally uniform, it will not necessarily remain so. The pressure due to solar radiation may thus through its selective action become a powerful factor in the disintegration of comets. Is it possible that we have here an explanation of the anomalous behaviour of Biela's comet? If from any cause or combination of causes the constituent particles became separated into two distinct sizes, the division of the nucleus follows as a natural consequence. The idea that the dimensions of the meteoric constituents were wanting in stability is supported by the remarkable variations in the relative brightness of the two nuclei and by the ultimate disappearance of the comet, which may have been due either to its transformation into a meteor swarm in the ordinary sense, and its consequent loss of brilliancy, or to its final disintegration. Another inference is suggested by the possibility that the effective attraction of the Sun on a comet is not necessarily constant. In order to disprove the identity of origin of two comets it does not seem sufficient to show that their elements do not satisfy Tisserand's criterion.

5. A further consequence of the radiation theory naturally presents itself as a plausible reason for the acceleration of the mean motion of a comet, and seems to deserve careful examination although the result will be found to be a negative one. This is derived from the application of Doppler's Principle to the radiation pressure against a moving surface. The result found by Poynting (p. 548, corrected by the note, p. 550) is that the force acting against a spherical particle which moves with the velocity v produces a tangential retardation $\mathrm{T}v/r^2$, where

$$\mathrm{T} = \mathrm{S}b^2/4\mathrm{U}^2\rho a,$$

S being the solar constant at the Earth's distance, b, from the Sun, and U the velocity of light. The action is clearly precisely analogous to that of a resisting medium of the type commonly assumed, and will tend to accelerate the mean motion in the same way, although the physical nature of the action is entirely different. There is a corresponding diminution in the distance from the Sun, a numerical example of which may be taken from Poynting's paper: a sphere of radius 1 cm. and density 5·5, starting with the velocity of the Earth and at its distance from the Sun will move inwards $\frac{1}{10000}$ of its distance in about 20,000

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years. This may be otherwise expressed by saying that its period of revolution is diminished by 0·2 sec. a year. If the same laws were applicable to a body actually as large as the Earth, the result would be to diminish the length of the year by 1 second in 3×10^9 years. The concrete question suggested by such results is whether the effect can reach an order which will account for the anomalous features which have been observed in the motion of Encke's comet.

6. Since T is constant for all distances and of an order such that its first power alone need be retained, the perturbations of an elliptic orbit, being entirely in the plane of motion, can be calculated very simply. The equations of motion can be written

$$\ddot{r} - r\dot{\theta}^2 = -\frac{\mu-\mu'}{r^2} - \frac{\mathrm{T}}{r^2}\dot{r}$$

$$\frac{d}{dt}(r^2\dot{\theta}) = -\mathrm{T}\frac{d\theta}{dt}$$

Integrating the latter and then eliminating t from the former, we get

$$r^2\dot{\theta} = \mathrm{T}/c - \mathrm{T}\theta$$

$$\frac{d^2u}{d\theta^2} + u = \frac{(\mu-\mu')c^2}{\mathrm{T}^2(1-c\theta)^2}$$

where u is the reciprocal of r, and c is an integration constant of the same order of magnitude as T. Hence the last equation may be replaced by the approximate form

$$\frac{d^2u}{d\theta^2} + u = (\mu-\mu')\ (1+2c\theta)c^2/\mathrm{T}^2$$

of which the integral is

$$u = (\mu-\mu')\ [1 + e\cos(\theta-\gamma) + 2c\theta]c^2/\mathrm{T}^2$$

Now it is evident, on general grounds, that the departure from a purely elliptic orbit in any one revolution is small. The preceding equation may therefore be considered a valid representation, during a small number of revolutions, of the motion of a particle which would, in the absence of the effect now considered, describe the elliptic orbit

$$u = (\mu-\mu')\ [1 + e\cos(\theta-\gamma)]c^2/\mathrm{T}^2$$

7. For the sake of definiteness and simplicity we may consider the motion at successive returns to the position which corresponds to the perihelion in the undisturbed orbit and put $\gamma = 0$. After k returns the motion is given by

$$u = (\mu-\mu')\ [1 + e\cos\theta + 2c(2\pi k + \theta)]c^2/\mathrm{T}^2$$

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where θ increases from the value 0. The osculating orbit at the point $\theta = 0$ is therefore

$$u = (\mu - \mu')\left[1 + e\cos\theta + 2c(2\pi k + \sin\theta)\right]c^2/\mathrm{T}^2$$

for this is an ellipse with its focus at the Sun and possessing the same circle of curvature as the disturbed orbit; and since the disturbing force has no normal component, equality of curvature implies equality of velocity in this ellipse and in the actual orbit. Writing the equation in a form in which the meaning of the constants is obvious, namely

$$lu = 1 + e'\cos(\theta - \beta)$$

we have

$$l^{-1} = (\mu - \mu')(1 + 4\pi ck)c^2/\mathrm{T}^2$$
$$\tan\beta = 2c/e$$
$$e'/l = (\mu - \mu')(e^2 + 4c^2)^{\frac{1}{2}}c^2/\mathrm{T}^2$$

These give the increments of the constants at each revolution. The position of the apse-line is constant and the variations of l and e' (of which the accent becomes superfluous) are given by

$$\frac{\delta l}{l} = \frac{\delta e}{e} = -4\pi c$$

Since $l = a(1 - e^2)$, we find

$$\frac{\delta a}{a} = -\frac{1 + e^2}{1 - e^2}\cdot 4\pi c$$

which leads further, since $n^2a^3 =$ constant, to

$$\frac{\delta n}{n} = \frac{1 + e^2}{1 - e^2}\cdot 6\pi c$$

8. The last result can be verified by a direct calculation of the periodic time P after k revolutions. For

$$\begin{aligned}
\mathrm{P} &= \int_{2k\pi}^{2(k+1)\pi} \frac{c.d\theta}{u^2\mathrm{T}(1 - c\theta)} \\
&= \frac{\mathrm{T}^3}{c^3(\mu - \mu')^2}\int_{2k\pi}^{2(k+1)\pi} \frac{d\theta}{(1 + e\cos\theta + 2c\theta)^2(1 - c\theta)} \\
&= \frac{\mathrm{T}^3}{c^3(\mu - \mu')^2}\int_{2k\pi}^{2(k+1)\pi} \left\{\frac{1}{(1 + e\cos\theta)^2} - \frac{c\theta(3 - e\cos\theta)}{(1 + e\cos\theta)^3}\right\} d\theta \\
&= \frac{2\mathrm{T}^3}{c^3(\mu - \mu')^2}\left\{\mathrm{I}_2 - \pi c(2k + 1)(4\mathrm{I}_3 - \mathrm{I}_2)\right\}
\end{aligned}$$

where

$$\mathrm{I}_5 = \int_0^{\pi} \frac{d\theta}{(1 + e\cos\theta)^5}$$

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Now

$$I_2 = \pi(1-e^2)^{-\frac{3}{2}}.$$
$$I_3 = \tfrac{1}{2}\pi(2+e^2)(1-e^2)^{-\frac{5}{2}}$$

Hence

$$P = \frac{2\pi T^3}{c^3(\mu-\mu')^2}(1-e^2)^{-\frac{3}{2}}\left\{1-3\pi c(2k+1)\frac{1+e^2}{1-e^2}\right\}$$

This is the period in the $(k+1)$th revolution, and consequently the variation of the period is given by

$$\frac{\delta P}{P} = -6\pi c\frac{1+e^2}{1-e^2}$$

which verifies the result found for the mean motion.

9. Now T/c is twice the areal velocity; hence if q is the perihelion distance

$$T/c = q^2 n(1+e)^{\frac{1}{2}}(1-e)^{-\frac{3}{2}}$$

Therefore

$$\delta n = 6\pi T(1+e^2)(1+e)^{-\frac{3}{2}}(1-e)^{\frac{1}{2}}/q^2$$

Using $1{\cdot}75\times10^6$ as the value of S and 3×10^{10} as the value of U in the expression given for T (§ 5), we have

$$6\pi T = 9{\cdot}2\times10^{-15}b^2/\rho a$$

Hence

$$\delta n = 9{\cdot}2\times10^{-15}(1+e^2)(1+e)^{-\frac{3}{2}}(1-e)^{\frac{1}{2}}b^2/q^2\rho a$$

or

$$\rho a = 9{\cdot}2\times10^{-15}\frac{(1+e^2)(1-e)^{\frac{1}{2}}}{(1+e)^{\frac{3}{2}}}\cdot\frac{b^2}{q^2}\cdot\frac{1}{\delta n} \quad \dots \quad \dots \quad (1)$$

which is the formula (in C.G.S. units), giving ρa when δn is known. Moreover, by connecting the expressions for δe and δn in terms of c (§ 7), we have

$$\frac{\delta e}{e} = -\frac{2}{3}\cdot\frac{1-e^2}{1+e^2}\cdot\frac{\delta n}{n}$$

or putting $e = \sin\phi$

$$\delta\phi = -\frac{1}{3}\cdot\frac{\sin 2\phi}{1+\sin^2\phi}\cdot\frac{\delta n}{n} \quad \dots \quad \dots \quad (2)$$

10. To represent the conditions of Encke's comet it is sufficient to take $q = \frac{1}{3}b$, $e = 0{\cdot}85$, $\phi = 58°$, and $n = 1070''$ per day. In accordance with Backlund's results, derived from the more recent apparitions, we may take $0''{\cdot}068$ as the increment of the mean daily motion in a revolution. The result of substituting these numbers in (2) is to give $\delta\phi = -2''{\cdot}3$, while Backlund's corresponding result is $-2''{\cdot}4$. The agreement between the two

numbers is, however, only apparently remarkable, for its significance is scarcely greater than that of an arithmetical check. As Backlund has pointed out, the relation between the variations δn and $\delta\phi$ which arise from the action of a resisting medium is practically independent of the nature of the medium, and can be assumed to be known. Hence, the action of the disturbing force here considered being formally identical with that of a resisting medium of simple type, it follows that the agreement is in reality nothing more than ought to be expected.

11. By expressing the above value of δn with a second as the unit of time and reducing to circular measure, we obtain

$$\delta n = 1/2{\cdot}62 \times 10^{11}$$

Hence the formula (1) gives

$$\begin{aligned}\rho a &= 9{\cdot}2 \times 10^{-15} \times 0{\cdot}265 \times 9 \times 2{\cdot}62 \times 10^{11}\\ &= 5{\cdot}7 \times 10^{-3}\end{aligned}$$

This shows that if we assume the density to be about equal to the mean density of the Earth a particle whose radius is one-hundredth of a millimetre and whose orbit round the Sun is identical with that of Encke's comet will have its mean motion accelerated by the same amount as has actually been determined from observations at the recent apparitions of the comet. But it does not follow from this that the periods of the comet and the particle will be the same. It is necessary to consider the effect of the direct radiation pressure. Now, by using the formula for μ' (§ 2) the above value of ρa gives

$$\mu'/\mu = 0{\cdot}013$$

so that the radiation pressure amounts to one-eightieth of the force due to solar gravitation. If the effect of this reduction in the solar attraction be attributed to errors in the assumed mean motion and mean distance, the relation between such errors is

$$\frac{2\Delta n}{n} + \frac{3\Delta a}{a} = \frac{\Delta\mu}{\mu} = -0{\cdot}013$$

On the assumption that the proportional errors in both elements are the same, each would be of the order 1 in 400. This implies an increase of the period by three days, which is scarcely possible! Nor is it easy to believe that the difficulty can be surmounted by the hypothesis of a sufficiently large error in the adopted mean distance. Consequently the difficulty of reconciling the suggested theory with the presumable state of the facts seems to be too great, and unless some serious error has been overlooked the attempt to explain the acceleration in the mean motion of Encke's comet by this particular deduction from the theory of radiation must apparently be regarded as an interesting but untenable speculation.

12. This, however, is not a conclusion from which there is definitely no escape. We are not so much seeking for a cause of the known peculiarity of the motion of Encke's comet as looking to that peculiarity for evidence of an actual physical process. Hence it seems unwise to dismiss hastily a theory in some ways so attractive. It may be possible, for instance, that the diminution in the solar attraction is neutralised by some other action such as electrostatic attraction. And apart from this possibility there is a consideration of a different kind. We have assumed a meteoric constitution for the comet, and it is difficult to avoid some such theory because the notion that a comet is composed of continuous matter seems impossible. Being widely dispersed the meteors must be practically unaffected by their mutual attractions, and may be considered to describe independent orbits. It is to *each member* of the total aggregate of orbits that the foregoing investigation applies, while the observations and the elements based on them refer to an object resulting indeed from the same aggregate, but connected with the individual members in a vague and undefined manner. Since we cannot observe each distinct meteor and trace its relative path within the swarm it is not necessary to assume that the mean distance which is deduced from the observations, and which has a significance purely statistical and quite possibly misleading, is identical with the mean distance of any particular meteor or the true average of the total number. This distinction, for which the theory of waves presents some analogy in the difference between group-velocity and wave-velocity, may be illustrated by the simplest possible example: if two particles describe similar coplanar orbits differing only in the fact that their axes are inclined to one another at an angle 2ϵ, the point midway between them also describes a similar orbit, but with a mean distance which bears to the mean distance of each particle the ratio $\cos\epsilon : 1$. The difficulty of assigning a definite dynamical meaning to the point to which the observations refer is further emphasised by the systematic differences which have been found to exist between observations made with large and with small telescopes. The great obstacle in the way of a satisfactory study of the statistical conditions lies in the difficulty of formulating an appropriate hypothesis regarding the complex motions of the constituents of the swarm.

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On the Validity of Meteor Radiants deduced from Three Tracks.

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(*Communicated by L. N. G. Filon, M.A., D.Sc.*)

1. It is not uncommon in tables of radiants of meteors to find radiants deduced from very few observations: four is quite a common number, and three by no means rare. We see, however, that it is quite possible that three or four paths, although quite unconnected, should happen to pass so closely through the same point that they would be taken to indicate a radiant there. It therefore becomes of interest to determine the probability of this happening. Suppose the mean divergence of an observed path from the radiant is ω, then it is clear that if the inradius of the Δ formed by the paths of three meteors $\not> \omega$ the three paths will be considered to pass through the same point and, subject to certain conditions which will be investigated later, to indicate a radiant there.

The problem before us is, therefore, to find the chance that the incircle of the Δ formed by three great circles taken at random on a sphere $\not> \omega$.

2. A uniform distribution of great circles on a sphere corresponds to a uniform distribution of their poles; accordingly we must have the circles so placed that it is possible to find a pole of each forming a Δ of circumradius $> \frac{\pi}{2} - \omega$.

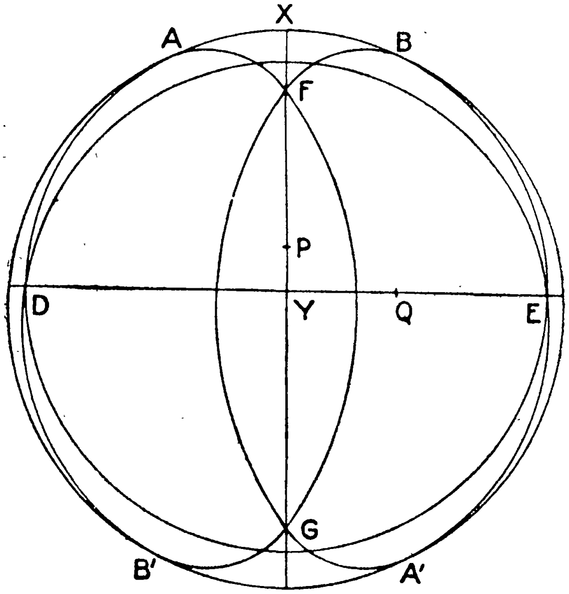

FIG. 1.

Let A (fig. 1) be a pole of one of the circles, and B that pole of the second which is nearer to A.